Burdett, R., Kozan, E., & Kenley, R.
(2015)
Block models for improved earthwork allocation planning in linear infrastructure construction.
Engineering Optimization, *47*(3), pp. 347-369.

Block Models for Improved Earthwork Allocation Planning in Linear Infrastructure Construction

R. Burdett, E. Kozan
School of Mathematical Sciences, Queensland University of Technology, Australia

R. Kenley
Faculty of Business and Enterprise, Swinburne University of Technology, Melbourne, Australia

Earthwork planning has been considered in this article and a generic block partitioning and modelling approach has been devised to provide strategic plans of various levels of detail. Conceptually this approach is more accurate and comprehensive than others, for instance those that are section based. In response to environmental concerns the metric for decision making was fuel consumption and emissions. Haulage distance and gradient are also included as they are important components of these metrics. Advantageously the fuel consumption metric is generic and captures the physical difficulties of travelling over inclines of different gradients, that is consistent across all hauling vehicles. For validation, the proposed models and techniques have been applied to a real world road project. The numerical investigations have demonstrated that the models can be solved with relatively little CPU time. The proposed block models also result in solutions of superior quality, i.e. they have reduced fuel consumption and cost. Furthermore the plans differ considerably from those based solely upon a distance based metric thus demonstrating a need for industry to reflect upon their current practices.

Keywords: mass-haul operations, earthwork allocation, fuel consumption metrics, block models.

1. Introduction

Earthworks are necessary in many infrastructure projects. In this article earthworks that are performed in linear infrastructure construction (such as road and rail) have been considered. These can be the source of very large earthworks as many have significant length and pass through difficult terrain. The principal earthwork operations are: i) stripping vegetation and topsoil, ii) loosening material in cutting and borrow pits, iii) excavating material, iv) loading material from cuts and hauling to fills (or to spoil), v) spreading, shaping, watering, compacting and trimming the fill material (*Queensland Transport and Main Roads* (QTMR), 1977). There are two golden rules that must be borne in mind during these projects; don't double handle material whenever possible and always load and carry material downhill (QTMR 1977).

The earthwork problem considered is commonly referred to as mass-haul. It involves the movement of material from one location to another, to alter an existing land surface, into a desired configuration (see Amar, 2003). Put most simply, it is the problem of strategically determining what earth goes where. To our understanding no earthworks project is performed without at least some form of rudimentary plan of the aforementioned type. In projects as large as these it could be catastrophic to act without some initial plans. Moving material to the wrong place, in the wrong amount, at the wrong time could be very costly.

Mass haul problems typically contain the full spectrum of decision making activities, namely sequencing and scheduling, assignment, selection and routing. Recent articles in this field include Son et al. (2005), Aruga et al. (2005), Akay (2006), Karimi et al. (2007), Kim et al. (2007), Goktepe et al. (2008), Zhang (2008), Dawood and Castro (2009), Ji et al. (2010), Hola and Schabowicz (2010), Ji et al. (2011), Shah and Dawood (2011), Nassar and Hosney (2012), Burdett and Kozan (2013a). Though much research has been performed in the last ten years, there are many limitations and inaccuracies in this work, and much of it is not comprehensive or detailed enough to be readily applicable to real life. A comprehensive review and critical analysis of this field has shown that there are many opportunities and avenues for future research. Foremost are integrated models that combine several decision-making problems. These include combined alignment design and earthwork allocation, combined earthwork scheduling and resource allocation, combined earthwork allocation and scheduling and resource allocation.

To improve earthworks several approaches have been developed for constructing superior *earthwork allocation plans* (EAP). These determine how the terrain is best altered to create a desired road (or other) surface by strategically determining where cut material is to be placed as fill and how much material is to be moved. This is not a trivial task in large projects. Hence this article concerns the development of improved methods and models. The advocated approach involves the partitioning of the domain into blocks and the solution of various mathematical programming models (i.e. LP, MIP). The selection of machines (i.e. for excavating, loading, and hauling) is a significant component of earthworks (i.e. in terms of operating cost), however this related task is not explicitly considered as it has been considered in other articles. Its integration is a source of future research. However, criterion governing what type of vehicle is used to perform hauls is incorporated. In that respect one aspect of resource selection has been integrated, in an indirect manner. The merit of an EAP has typically been judged by its total haul distance and to a lesser extent by the total haul time (Son et al, 2005). In other words the relative merit of moving material from one area to another is based on the distance of the haul or the time to make the haul. The approximated distance is typically calculated between the middle points of predefined sections, in one or two dimensions (Son et al, 2005). Distance and time metrics however are quite approximate, and do not include land gradients or other important factors. For example at different angles the force retarding or enhancing motion is different. Therefore in many situations a long downward path is more desirable than short level trips. As a consequence of these observations the physics concept work has been selected as the primary metric for decision making. It is relatively generic and straightforward to use. It captures the effect of distance and gradient which are predominant in the cost of hauls, and whose effect is consistent across all hauling vehicles.

In recent years, the environmental impact of construction activities has become increasingly important due to the pollution that is created. The quantification of *green house gas emissions* (GHGE)) is particularly important given the current political situation and the environmental concerns of people in many countries over climate change. The possibility of measuring GHGE in real time has been reported by Lee et al. (2009). They proposed the use of a web based management system and a wireless network to record measurements on construction sites, and to implement an emissions trading scheme. Kim et al (2012) and Lewis and Hajji (2012) have recently provided methodologies and frameworks for the calculation of GHGE in road construction and in earthworks. Their approach like many others is generic and is based upon empirical results and simple formulas; these however do not necessarily provide accurate results in all situations and are based upon many assumptions. Carmichael et al (2012) considered the relationship between operating costs and emissions in earthmoving operations. The equivalence between the minimum cost per production and minimum emissions per production solutions was demonstrated for a simple case study and for a set of assumptions and configurations. From this result it was concluded that current practices for earthmoving reduce the environmental impact and need no modification. Further investigation however is necessary to prove that this is universally true.

The literature also suggests that GHGE in mining are indicative of those in mass haul problems. Norgate and Haque (2010) have found that loading and hauling operations contributed the most in mining and mineral processing (roughly 50%). Kim et al (2012) have similarly found that earthworks produced the greatest emissions in construction operations and dump trucks (i.e. the hauling units) were the greatest source of emissions, with bulldozers and loaders being the next biggest polluters. Proper selection of equipment was described as a strategy for reducing emissions. To this end Avetisyan et al (2012) developed an optimization approach for equipment selection that minimizes emissions. In Lewis and Hajji (2012) it has been reported that as the soil type becomes more difficult to excavate, the production rate of machines decreases and the total activity duration increases, thus more fuel is consumed and more CO2 is emitted. Hence where roads and other infrastructures are built is of great importance.

In response to the aforementioned research and the recent introduction of a carbon tax in Australia, fuel consumption metrics have been introduced in this article. These allow emissions resulting from earthworks to be computed. It should be noted that the proposed fuel consumption metrics are proxies, as in many (but not all) circumstances, it is impractical to measure or quantify the exact fuel consumption of every vehicle used on a construction site and for every associated factor that affects fuel consumption (the predominant factors being mass, speed of travel, age of machine, engine type/power, fuel type, angle of travel, temperature and terrain type). For the purposes of this article, and for ease of use by contractors, a generic and perhaps more approximate fuel consumption metric is more valuable than a more accurate but complex metric (or process). It should also be pointed out that an earthwork allocation is a plan of one aspect of earthwork activities and does not necessarily include the types of vehicles that will be used in the project; these are often unknown and or variable. Consequently there is little point in creating an earthwork allocation based upon the specific fuel consumption attributes of one fleet of vehicles if the fleet is changed numerous times before and during construction activities. To quantify the total emission level (and in the absence of any other information to the contrary) fuel consumption (in litres) is multiplied by a static parameter for the emission per litre of fuel consumed. Last it should be mentioned that the reduction of fuel consumption is very important in its own right. Recent site visits and consultation with project management on the large Bruce Highway Upgrade at Gympie in Queensland (see Qld Roads 2011), has verified that there is tens of thousands of dollars worth of fuel (if not more) sitting in vehicles on project sites.

Generic measures of fuel consumption have been utilised in this article, but a number of other approaches are available, and should be used where applicable. The modelling of fuel consumption is not a new topic, and the literature has many articles on this topic. Previous research has often concentrated on determining accurate models for specific vehicle types such as cars (see Post et al. (1984), Akcelik (1989), Akcelik 2012)). These approaches are based upon real consumption patterns of considered vehicles, and are rarely conceptual. They include things like driving patterns and stopping and acceleration phases. One popular approach is to model fuel consumption by fitting regression models to observed data. To our understanding only a few of the main factors affecting fuel consumption are specifically included. In addition the results are often described as merely satisfactory; implying a significant level of inaccuracy exists. Other previous approaches are somewhat unrealistic for vehicles used in construction as they are based upon driving in an urban setting, which is over smooth stable terrain and shallow gradients. Cars also have to brake regularly as they need to interact with other vehicles, traffic lights and speed restrictions.

In the next section, a section based approach is considered for constructing an EAP and a mathematical model is presented. In section 3 a block modelling approach is introduced and two alternative mathematical models are developed. Fuel consumption metrics are then proposed in section 4. These are vital components in the decision making activities of previous sections. A case study is described in section 5 upon which the models of this article are applied. The conclusions, future research directions and final remarks are then given to conclude the article.

2. Section Based Models

2.1. *Sections and Distances*

A typical mass-haul problem is described by two “longitudinal” land profiles. One is for the ground as it currently is and one is for the planned surface (i.e. road). Each profile is a line chart that shows the elevation of the land (in the z-axis) at specified stations (i.e. chainages) along the x-axis. The two profiles can intersect at various locations (i.e. intersection points) and these “points” generally distinguish where cutting and filling activities end. For each coordinate in the y-axis, there is an equivalent longitudinal profile. As roads and other linear infrastructure are narrow it is sometimes assumed that the elevations remain the same across the y-axis. The removal and replacement of inferior or contaminated material (below the aforementioned profiles) is also typical of many projects, but these volumes must be defined differently.

The distance between specified locations is defined as a section. The distance between adjacent intersection points is an alternative definition. A significant difference between the two alternatives is that sections only require cutting or filling in the second approach. However a limitation of the second approach is a lack of control over how many sections are involved and the distance between each. In previous research it has been assumed that material movement occurs between section mid points and the distance of the haul is the difference in the associated chainages. Consequently haul distances become more inaccurate as the section length increases. In this article the distance between two locations is approximated by the line(s) connecting the two elevations at those locations. This means that movement occurs through the terrain as it currently is. This practice has been verified by industry. Therefore more accurate haul distances can be computed and gradient information can be incorporated. This is necessary when quantifying fuel consumption and emissions. This policy also keeps trucks and other vehicles off main roads and limits unnecessary and negative interaction with the community. Larger and more capable machinery can then be used on project sites that are more productive. From an environmental perspective it is crucial that surrounding areas remain untouched.

A section redefinition that reinstates linearity of land profiles for each section is desirable. A combination of the previous strategies was proposed to facilitate this. That is, both the original locations plus the locations of intersection points are used to partition the domain. Additional locations and sections could also be added to improve the accuracy of modelling activities; the only requirement is that each section is described by a linear profile and that cutting only or filling only activities occur. In our opinion section lengths should not be excessively large or too small however this is open to debate and other practical considerations.

2.2. *Notation*

The notation used to describe the first model for constructing an EAP is now introduced. Conceptually the main components are locations and sections. Some parameters have been used for both locations and sections. The use of index *i* and *j* refers to the former, while the use of *u* and *v* the later. Each location along the x-axis has a position. This is often referred to as the chainage and is denoted by x_i. The elevation of the land at location *i* is denoted as h_i. Each section is bounded by two locations. The left and right boundary of section *u* is denoted by l_u and r_u respectively. This is an index and not a chainage. Along the x-axis the section middle point is $x_u^{\mathrm{mid}} = (x_j + x_i)/2$ where $(i,j) = (l_u, r_u)$. It's elevation is $h_u^{\mathrm{mid}} = (h_j + h_i)/2$. The gradient within each section is $g_u = (h_j - h_i)/|x_j - x_i|$ (when travelling from *i* to *j*) and $g_u = (h_i - h_j)/|x_i - x_j|$ (when travelling from *j* to *i*). The section gradient is only valid given the assumption of a linear land profile. The symbol *g* is also used to denote the force of gravity in section 4. It is not included in any other part of this article and it should be clear which parameter is being referred to. When referring to the attributes of a planned surface (like a road), as opposed to the current terrain, an over bar is used, i.e. $\bar{h}_i$, $\bar{h}_u^{\mathrm{mid}}$, $\bar{g}_u$. Therefore, $\bar{h}_u^{\mathrm{mid}} = (\bar{h}_j + \bar{h}_i)/2$ and depending on direction of travel: $\bar{g}_u = \frac{\bar{h}_j - \bar{h}_i}{|x_j - x_i|}$ or $\bar{g}_u = \frac{\bar{h}_i - \bar{h}_j}{|x_i - x_j|}$.

The distance between adjacent locations and sections is denoted as $d_{i,j}^{\mathrm{L}}$ and $d_{u,v}^{\mathrm{S}}$ respectively. The superscript is used to distinguish between distance calculations involving locations and sections and is otherwise redundant. The distance travelled between non adjacent locations and sections, is denoted by $D_{i,j}^{\mathrm{L}}$ and $D_{u,v}^{\mathrm{S}}$ respectively. When the profile of the land is approximated by line segments, the Euclidean (direct) distance between two locations *i* and *j* is calculated in the following way:

$$d_{i,j}^{\mathrm{L}} = \sqrt{\left(x_j - x_i\right)^2 + \left(h_j - h_i\right)^2} \qquad [i \text{ and } j \text{ adjacent}]$$

$$D_{i,j}^{\mathrm{L}} = \sum_{l=i}^{j} \sqrt{(x_l - x_{l+1})^2 + (h_l - h_{l+1})^2} = \sum_{l=i}^{j} d_{l,l+1}^{\mathrm{L}} \qquad [i \text{ and } j \text{ not adjacent}]$$

The distance between section *u* and *v* is taken as the distance between the mid points on the land profile and is the sum of two line segments; it is not taken as the Euclidean distance.

$$d_{u,v}^{\mathrm{S}} = \sqrt{\left(x_j - x_u^{\mathrm{mid}}\right)^2 + \left(h_j - h_u^{\mathrm{mid}}\right)^2} + \sqrt{\left(x_j - x_v^{\mathrm{mid}}\right)^2 + \left(h_j - h_v^{\mathrm{mid}}\right)^2} = \left(d_{i,j}^{\mathrm{L}} + d_{j,k}^{\mathrm{L}}\right)/2$$

where $(i,j) = (l_u, r_u)$ and $(j,k) = (l_v, r_v)$. For non adjacent sections, the distance is computed as: $D_{u,v}^{\mathrm{S}} = \sum_{\alpha=u}^{v}\left(d_{\alpha,\alpha+1}^{\mathrm{S}}\right)$. It should be pointed out that the above distance calculations are for locations and sections within the project site. For borrow and disposal sites the values of $D_{\bar{u},v}^{\mathrm{S}}$ and $D_{u,\bar{v}}^{\mathrm{S}}$ must be given; they are not calculated per se. It should also be noted that the above distances are measured in terms of the current state of the land. However once construction begins, many of those distances are no longer accurate due to changes in the terrain from cuttings and filling. Measuring distances according to the final surface is also flawed because construction and haulage does not occur across that profile. One possible solution is to average out the two profiles. For example the average height (i.e. halfway point) between the planned surface and the actual terrain could be used. That is, replace all values of h_i with $\bar{\bar{h}}_i$ where $\bar{\bar{h}}_i = \left(h_i + \bar{h}_i\right)/2$ and $h_u^{\mathrm{mid}} = \left(\bar{\bar{h}}_j + \bar{\bar{h}}_i\right)/2$. This approximation is shown in Figure 1 and will be closer "on average" to the actual situation encountered in practice. The gradient of the section (i.e. when travelling left to right) is also redefined as follows: $\bar{\bar{g}}_u = \frac{\bar{\bar{h}}_j - \bar{\bar{h}}_i}{|x_j - x_i|}$.

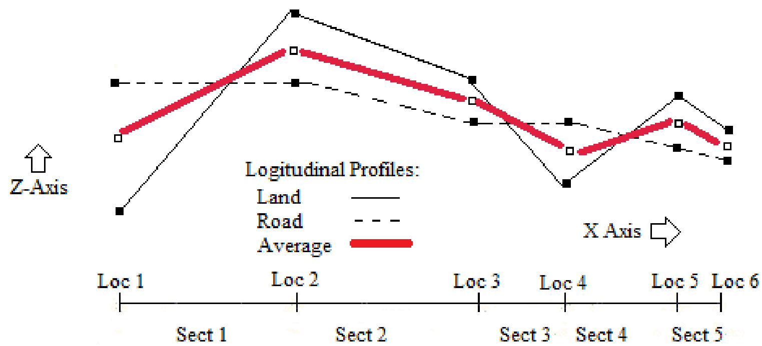


Figure 1. Creation of an average longitudinal profile

2.3. *Section Allocation Model*

The decision variable for this model is denoted by $Q_{u,v,s}$ and describes the volume to be cut from section *u* of soil type *s* and moved to section *v*. Sections are partitioned into four sets, namely: $\mathrm{Cut}, \mathrm{Fill}, \mathrm{Borrow}, \mathrm{Waste}$. Borrow and waste sites are also regarded as sections. The set of cut section include the borrow site(s) as these are sources of cut material too. Similarly the set of fill sections include the waste site(s) as these are destinations for cut material. The amount (i.e. volume) of cut and fill of soil type *s,* in each section, is denoted by $qty\_cut_{u,s}$ and $qty\_fill_{v,s}$ respectively. These parameters also define capacity limitations on borrow and waste sites; they are not strict cut and fill requirements as such. The contraction/expansion of soil as a result of either excavations, compaction (i.e. filling) and transportation are described respectively by the parameters γ_s^{E}, γ_s^{F} and γ_s^{T}. They refer to the percentage difference in volume that occurs. The aggregation of these values is as follows, $\Upsilon_{\mathrm{s}} = \gamma_s^{\mathrm{E}}\gamma_s^{\mathrm{F}}\gamma_s^{\mathrm{T}}$, and refers to the combined effect of the three activities. The emission per litre of fuel consumed is Ω. The fuel consumption for haulages between adjacent sections is denoted as $f_{u,v}$. It's calculation is discussed later. For non adjacent sections: $F_{u,v} = \sum_{\alpha=u}^{v}(f_{\alpha,\alpha+1})$. The proposed model is as follows:

Minimise: $EM = \Omega \times \sum_{s}\left(\sum_{u\in\text{Cut}}\sum_{v\in\text{Fill}}\left(Q_{u,v,s} \times F_{u,v}\right)\right)$
Subject to:

$$\sum_{v\in\text{Fill}}\left(Q_{u,v,s}\right) = qty\_cut_{u,s} \qquad \forall u \in \text{Cut}|u \notin \text{Borrow}, \forall s \in \text{Soil} \tag{1}$$
$$\Upsilon_s \times \sum_{u\in\text{Cut}}\left(Q_{u,v,s}\right) = qty\_fill_{v,s} \qquad \forall v \in \text{Fill}|v \notin \text{Waste}, \forall s \in \text{Soil} \tag{2}$$
$$Q_{u,v,s} \geq 0 \qquad \forall u \in \text{Cut}, \forall v \in \text{Fill}, \forall s \in \text{Soil} \tag{3}$$
$$\Upsilon_s \times \sum_{u\in\text{Cut}}\left(Q_{u,v,s}\right) \leq qty\_fill_{v,s} \qquad \forall v \in \text{Waste}, \forall s \in \text{Soil} \tag{4}$$
$$\sum_{v\in\text{Fill}}\left(Q_{u,v,s}\right) \leq qty\_cut_{u,s} \qquad \forall u \in \text{Borrow}, \forall s \in \text{Soil} \tag{5}$$

Constraint (1) ensures that the material removed from each section must equal the defined amount of cut. Similarly (2) ensures that the material brought to a section must equal the defined amount fill. Constraint (3) ensures that the amount of material moved between sections must be positive. Constraint (4) and (5) are capacity constraints; they ensure that the waste/borrow sites respectively are not over utilised. In the objective function, the fuel consumption is computed over all movements and then multiplied by the emission parameter. The fuel consumption that occurs between each section is multiplied by the total amount of material hauled. This is an inflated value and is not the real fuel consumption. When vehicle information is available this approach should be updated. For example, to obtain the real fuel consumption, the number of times the journey between section *u* and *v* is made, must be incorporated. If the capacity of the hauling vehicle is $\hat{Q}_s$ cubic metres then $\lceil Q_{u,v,s}/\hat{Q}_s \rceil$ is the number of hauls and this value should replace $Q_{u,v,s}$ in the objective function.

In the event that numerous soil types are acceptable as fill within sections, a binary parameter denoted by $\kappa_{v,s}$ ($\forall v \in \text{Fill}$) can be introduced. It equals one if material of type *s* is acceptable in section *v*, and zero otherwise. The following constraints are then added to the model.

$$\sum_{s}\left(\Upsilon_s \times \sum_{u\in\text{Cut}}\left(Q_{u,v,s}\right)\right) = qty\_fill_{v} \qquad \forall v \in \text{Fill}|v \notin \text{Waste} \tag{6}$$
$$\sum_{s}\left(\Upsilon_s \times \sum_{u\in\text{Cut}}\left(Q_{u,v,s}\right)\right) \leq qty\_fill_{v} \qquad \forall v \in \text{Waste} \tag{7}$$
$$Q_{u,v,s} \leq \kappa_{v,s}\mathcal{L} \qquad \forall u \in \text{Cut}, v \in \text{Fill}, s \in \text{Soil} \tag{8}$$

Constraint (6) and (7) in particular replace constraint (2) and (4). They recognise that a "general" volume is required and it could be made up of many different soil types. Constraint (8) rejects the movement of invalid material types according to the binary parameter. Note that $\mathcal{L}$ represents a sufficiently large number.

In practice the determination of soil types is via bores. They are drilled into the ground well below planned surfaces. The soil types at that location, over that depth, is then determined. In theory the ground could radically change even within the vicinity of a bore. Hence in that scenario inaccuracies may be introduced in some projects. It should however be noted that all earthworks planning suffers from inaccuracies of one form or another as complete and "perfect" information is impossible to obtain (i.e. because of measurement errors, etc). Given uncertainties concerning soil types, the idea that the soil at each "cut" section is suitable (good enough) or not suitable (not good enough) as fill has been considered. Similarly it may be necessary to define (approximate) the actual percentage that is good. This percentage is denoted by $suit_u$. By altering $suit_u$, a sensitivity analysis can be made of allocation solutions or a revised solution can be created in the event of more accurate on-site information. In this simplification there will be two soil types, namely, suitable (used as fill) and unsuitable (dumped as waste). The associated cut amounts are therefore: $cut_{u,1} = \frac{suit_u}{100} \times vol_u$ and $cut_{u,2} = \left(\frac{100-suit_u}{100}\right) \times vol_u$ $\forall u \in \text{Cut}$. Similarly, $fill_{v,1} = vol_v$ and $fill_{v,2} = 0$ $\forall v \in \text{Fill}$. It should be noted that borrow sites are assumed to contain only suitable material, i.e.

$suit_u = 100\ \forall u \in \text{Borrow}$. Waste sites however can contain both suitable and unsuitable material and the percentage split must be given.

The traditional metric of previous models is to minimise the total movement of all material, where $D^{\text{S}}_{u,v} = x^m_v - x^m_u$. Given the distance redefinition described earlier in the paper, that metric becomes the following, $TKM = \sum_s \sum_{u\in\text{Cut}} \sum_{v\in\text{Fill}} \left(Q_{u,v,s} \times D^{\text{S}}_{u,v}\right)$ where TKM is an abbreviation for tonne kilometres. However in this model fuel consumption and emissions are of more interest.

The main innovation of the first model is that gradient and fuel consumption have been incorporated, and soil types can be dealt with in several ways. However as with the majority of all other models, the distances are assumed to be static and do not reflect the real terrain after cutting and filling activities have begun. It was earlier mentioned that fuel consumption is pre-calculated. In standard situations (i.e. for travel between adjacent sections) it is calculated independently in the following way:

$$f_{u,v} = \mathbf{FUEL}\left(M, d^{\text{L}}_{i,j}/2, \bar{\bar{g}}_v\right) + \mathbf{FUEL}\left(M, d^{\text{L}}_{j,k}/2, \bar{\bar{g}}_v\right) \text{ where } (i,j,k) = (l_u, r_u, r_v)$$

where **FUEL** is a function that returns for a vehicle of a given mass, travelling for a given distance, over a given gradient, the fuel consumed. In Section 4, different fuel consumption metrics are proposed for this function. These functions also rely upon the vehicles speed and cross sectional area. The choice of *M* is problematic in some instances, as is the selection of *A* and *V* (i.e. the cross sectional area and the speed of travel). In most situations the vehicle that performs the majority of the hauls is the most obvious choice for deciding the appropriate values. In reality, many different vehicles are used by contractors. The vehicles are hired for different periods of time and those on site change from week to week. On large projects, insufficient machinery may be available, for instance in remote sites. This has been verified by industry (i.e. at the Bruce Highway upgrade at Gympie in Queensland). On other occasions an average vehicle mass may be appropriate, or even a conglomeration of several masses if different vehicles are used (i.e. as they commonly are). For instance, trucks, scrapers and bulldozers may all be used on a project.

The mass, cross-sectional area and speed may even be a function of the distance travelled and may be different for each pair of sections (i.e. cut-fill pairing). For instance the distance travelled could be used to distinguish which type of machinery is used. For example, if the distance is less than 50 metres then a bulldozer should be used. If the distance is less than 1500 metres but greater than 50 metres, a scraper should be used. For all other distances greater than 1500 metres, trucks should be used. If the distance is 3km or larger then highway trucks should be used else off highway trucks should be used. This logic is described by the following function:

$$\left(M_{u,v}, A_{u,v}, V_{u,v}, \hat{Q}_{u,v,s}\right) = \boldsymbol{\psi}\left(D^{\text{S}}_{u,v}\right) = \begin{cases} \left(M^1, A^1, V^1, \hat{Q}^1_s\right) & D^{\text{S}}_{u,v} \leq 50 \\ \left(M^2, A^2, V^2, \hat{Q}^2_s\right) & 50 < D^{\text{S}}_{u,v} \leq 1500 \\ \left(M^3, A^3, V^3, \hat{Q}^3_s\right) & 1500 < D^{\text{S}}_{u,v} \leq 3000 \\ \left(M^4, A^4, V^4, \hat{Q}^4_s\right) & D^{\text{S}}_{u,v} > 3000 \end{cases}$$

More accurate and elaborate conditions may be easily incorporated by extending this function. In practice there is no set rule regarding vehicle selection. Recent site visits to the Bruce Highway upgrade at Gympie (see Qld Roads 2011) has revealed that contractors will often do what is necessary at the time, based on material type and machine availabilities. For example sometimes the most suitable item of plant will not be available and a less satisfactory machine will have to be used. These practices are reflections of real operations, however, in no way nullify the benefits of proper planning or re-planning.

The capacity of the different vehicles are given by $\hat{Q}^1_s, \hat{Q}^2_s, \hat{Q}^3_s, \hat{Q}^4_s$. It is therefore necessary for the term $\left\lceil Q_{u,v,s}/\hat{Q}_s \right\rceil$ to be replaced with $\left\lceil Q_{u,v,s}/\hat{Q}_{u,v,s} \right\rceil$ in the objective function. Because of this extension the fuel consumption calculations between sections, must be revised. It should be noted that $f_{u,v}$ is normally static and it does not take into account longer hauls, that incorporate journeys between *u* and *v* within them. Consequently the following equation should be used to determine the fuel consumption of longer hauls:

$$F_{u,v} = \sum_{\sigma=u}^{v}\left(\mathbf{FUEL}\left(M_{u,v}, A_{u,v}, V_{u,v}, d_{i,j}^{\mathrm{L}}/2, \bar{\bar{g}}_{\sigma}\right) + \mathbf{FUEL}\left(M_{u,v}, A_{u,v}, V_{u,v}, d_{j,k}^{\mathrm{L}}/2, \bar{\bar{g}}_{\sigma}\right)\right)$$

In the above equation, $(i, j, k) = (l_{\sigma}, r_{\sigma}, r_{\sigma+1})$. The proposed fuel consumption metrics are based upon the forces retarding motion. This force is a function of *M*, *A* and *V*. These components can be easily ignored if a general effect is sufficient. One way to do this would be to set them to unity. Another would be to completely remove the drag term from the force term.

As the emissions factor is a scalar, the objective function is really considering the minimization of fuel consumption which (fortunately) is a significant component of total project cost. The project costs could be modelled even more accurately by "tallying" the time that each of the different vehicle types is needed in the EAP. This value could be converted to a number of days and then multiplied by an appropriate cost per day parameter to give an approximate operating cost.

3. Block Models

In mining problems, a "partitioning" of the problem domain into discrete rectangular prisms (3D blocks) is often used. Conceptually this approach is well suited to earthworks too and is investigated in this article. It should be pointed out that block structures are particularly amenable to more powerful and capable optimization and graph theoretic strategies.

In this block modelling approach, the set of blocks requiring excavation and fill respectively are denoted by (B^{-}, B^{+}). The set of borrow and waste site blocks are denoted respectively by $(\hat{B}^{-}, \hat{B}^{+})$. The set of all blocks is therefore, $B = B^{-} \cup B^{+} \cup \hat{B}^{-} \cup \hat{B}^{+}$. The size of each block *b* is specified as $(\Delta x, \Delta y, \Delta z)$ and its volume is denoted and calculated by $vol_b = \Delta x \times \Delta y \times \Delta z$. The volumes of borrow and waste blocks are not cut/fill requirements but capacities. If the material in each block is uniform then ω_b denotes the material (soil) type of block *b*.

The volume of cut and fill in each block are defined as cut_b and $fill_b$ respectively. The volume of each soil type is hence $cut_{b,s}$ and $fill_{b,s}$. Ordinarily, $B^{-} = \{b | \sum_s cut_{b,s} > 0\}$, $B^{+} = \{b | \sum_s fill_{b,s} > 0\}$ and blocks may be in both sets. In some blocks, certain material types may be acceptable or not acceptable as fill. A binary parameter that equals one if material of type *s* is acceptable in block *b*, and zero if material is not acceptable, is defined as $\kappa_{b,s}$.

The position of block *b* in three dimensional space is given by the grid location (x_b, y_b, z_b). Its middle point is $m_b = \left((x_b - 0.5)\Delta x, (y_b - 0.5)\Delta y, (z_b - 0.5)\Delta z\right)$. The distances, gradient, angle and fuel consumption associated between any two blocks b and b' are denoted respectively by $d_{b,b'}$, $g_{b,b'}$, $\theta_{b,b'}$ and $f_{b,b'}$. The direct distance between blocks is easily determined from midpoint to midpoint using either a Euclidean (direct) or rectilinear (sum of movements in each axis) distance metric as shown below:

$$d_{b,b'} = \sqrt{(x_b - x_{b'})^2 \Delta x^2 + (y_b - y_{b'})^2 \Delta y^2 + (z_b - z_{b'})^2 \Delta z^2}$$
$$d_{b,b'} = |x_b - x_{b'}|\Delta x + |y_b - y_{b'}|\Delta y + |z_b - z_{b'}|\Delta z$$

The rectilinear distance metric describes the total horizontal and vertical movements required. In some ways it is more indicative of the effort of excavating and hauling material from a given elevation. However this metric may over inflate the actual distance travelled when sloping paths are available.

Associated with direct measures of distance, between two blocks, are the gradient (i.e. the ratio of the vertical difference to the horizontal difference) and the angle of the associated incline. Between any two blocks these are calculated in the following way:

$$g_{b,b'} = [(z_{b'} - z_b)\Delta z]/\sqrt{(x_b - x_{b'})^2\Delta x^2 + (y_b - y_{b'})^2\Delta y^2}$$
$$\theta_{b,b'} = \tan^{-1}(g_{b,b'})$$

The fuel consumption that occurs when moving material between blocks is as follows:

$$f_{b,b'} = \mathbf{FUEL}(M_{b,b'}, A_{b,b'}, V_{b,b'}, d_{b,b'}, g_{b,b'},)$$

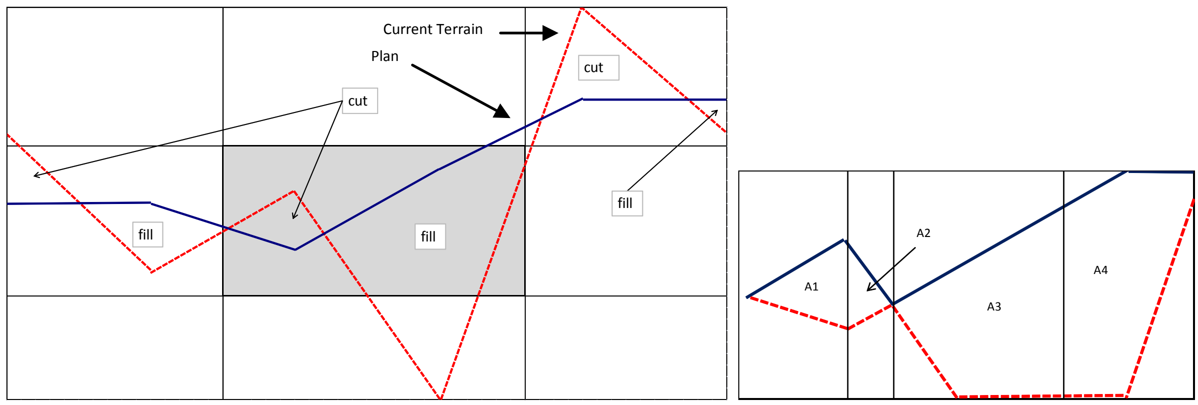
Using the approach in section 2.3, the vehicle mass, cross-sectional area and speed that should be associated with the movement of material between blocks *b* and *b'* is as follows:

$$(M_{b,b'}, A_{b,b'}, V_{b,b'}, \hat{Q}_{b,b'}) = \boldsymbol{\psi}(d_{b,b'}, \omega_b) = \begin{cases} (M^1, A^1, V^1, \hat{Q}^1_{\omega_b}) & d_{b,b'} \leq 50 \\ (M^2, A^2, V^2, \hat{Q}^2_{\omega_b}) & 50 < d_{b,b'} \leq 1500 \\ (M^3, A^3, V^3, \hat{Q}^3_{\omega_b}) & 1500 < d_{b,b'} \leq 3000 \\ (M^4, A^4, V^4, \hat{Q}^4_{\omega_b}) & d_{b,b'} > 3000 \end{cases}$$

3.1. *Block Partitioning*

Conceptually the partitioning of the domain into blocks is highly advantageous and provides a far greater level of accuracy than the section based approach of section 2. There are numerous ways to partition the domain. The most obvious is to choose a suitable sized block and then to uniformly place these blocks around the original domain. An alternative "transformed" domain can also be used. This transformed domain is created by subtracting the road elevations from the ground elevations (or vice versa). This approach is utilised in Shah et al. (2011), and forms the foundation of their approach.

Sloping ground and road profiles however cause some blocks to be intersected. Those blocks may be partially full if they occur at the boundary between land and sky or at the boundary of static terrain (i.e. uncut ground). Under the assumption that only whole blocks are modelled, a choice must be made as to whether the block is full or empty. This choice causes measurement errors and these are manifested as over/under cutting/filling. This may be acceptable in many projects, particularly if the block size is small. Without the help of more advanced software, the process of defining blocks is very "labour" intensive; particularly in 3D. Microsoft Excel is sufficient for small 2D problems with large block sizes. To automate the process of block definition (in 2D) computer algorithms have been implemented and these are recommended. The basis of these is to determine which blocks must be included in the problem, what volume of material occurs within the block, and whether the block is to be recognised as a cut, fill or cut and fill block. In the interest of brevity the exact procedures for partitioning and block identification have been omitted but are available from the authors upon request. The main logic of these algorithms is to separate the segmented ground and road profiles into a segmented top and bottom line. By analysing points on top and bottom lines in each segment within each block, the correct areas can be identified relatively easily. The number of segments is irrelevant and very complex areas can be identified. An example of the process is shown in Figure 2. Figure 2a shows an example of a road (solid line) and the current terrain (dotted ground line). The block in the middle has been highlighted for demonstration purposes. The cut and fill areas are computed by analysing four separate segments (sub blocks) as shown in Figure 2b. Note that lines are truncated when they pass outside the boundary of the block. Each of these areas can be defined as polygons and the area can be defined using an appropriate procedure for area determination. However, such an approach is unnecessary; the difference in the areas under each line provides the same result. The blocks volume is then the area multiplied by the road width.



a) A partitioned longitudinal profile (XZ axis) b) shaded block has four areas

Figure 2. Cut and fill area identification within blocks

3.2. *Block Allocation Model 1*

The first block model that is proposed assumes that soil type within each block is uniform and blocks are whole. That is each cut block is initially full of material and each fill block has no material. Therefore "whole" blocks are cut, moved, and placed as fill and a blocks material is not divided; it goes to one location. This is not an unrealistic assumption in many situations. From a practical perspective it is perhaps more realistic to dig up a discrete block of earth and to shift it to one specified place as opposed to trying to accurately break it up into many smaller parts and to send them to many specific locations. Alternatively if blocks are small enough, then cut and fill volumes will be close (if not exact) to the block's volume. Unlike other approaches, the amount of material to be moved is not necessarily a decision when using blocks in this way (but could be). Where material is to be exactly placed however is to be decided; this choice is discrete and finite. A binary decision variable denoted by $H_{b,b'}$ is defined to signify whether material is hauled from block b to b' (i.e. 1 means yes, 0 means no). The traditional metric of previous models, that is to minimise the total movement of all material, is computed in the following way: $HD = \sum_{b\in(B^-\cup\hat{B}^-)}\sum_{b'\in(B^+\cup\hat{B}^+)}\left(H_{b,b'}\times d_{b,b'}\right)$ where *HD* is an abbreviation for haul distance. This metric is used for comparison purposes in the later case study of this article. The following block optimization model is now proposed:

Minimise $EM = \Omega \times \sum_{b\in(B^-\cup\hat{B}^-)}\sum_{b'\in(B^+\cup\hat{B}^+)}\left(H_{b,b'}\times\left\lceil vol_b/\hat{Q}_{b,b'}\right\rceil\times f_{b,b'}\right)$

Subject to:

$$\sum_{b'\in(B^+\cup\hat{B}^+)}\left(H_{b,b'}\right)=1 \quad \forall b\in B^- \tag{9}$$

$$\sum_{b'\in(B^-\cup\hat{B}^-)}\left(H_{b',b}\right)=1 \quad \forall b\in B^+ \tag{10}$$

$$\sum_{b'\in(B^+\cup\hat{B}^+)}\left(H_{b,b'}\right)\le 1 \quad \forall b\in \hat{B}^- \tag{11}$$

$$\sum_{b'\in(B^-\cup\hat{B}^-)}\left(H_{b',b}\right)\le 1 \quad \forall b\in \hat{B}^+ \tag{12}$$

$$H_{b,b'}=0 \quad \forall b\in\left(B^-\cup\hat{B}^-\right),\forall b'\in\left(B^+\cup\hat{B}^+\right)\mid\omega_b\neq\omega_{b'} \tag{13}$$

$$H_{b,b'}\le\kappa_{b',\omega_b} \quad \forall b\in\left(B^-\cup\hat{B}^-\right),\forall b'\in\left(B^+\cup\hat{B}^+\right) \tag{14}$$

$$H_{b,b'}=0 \quad \forall b\in\hat{B}^-,\forall b'\in\hat{B}^+ \tag{15}$$

$$H_{a,d}+H_{b,c}\le 1 \quad \forall(a,b,c,d)\in\wp \tag{16}$$

$$H_{b,b'}\in\{0,1\} \quad \forall b\in\left(B^-\cup\hat{B}^-\right),\forall b'\in\left(B^+\cup\hat{B}^+\right) \tag{17}$$

In the objective function, the term $\left\lceil vol_b/\hat{Q}_{b,b'}\right\rceil$ defines the number of hauls required for the specified vehicle type to haul the blocks volume. Constraint (9) ensures that each cut block must be moved to a single fill block. Constraint (10) similarly ensures that each fill block must be filled by a single cut block. Constraint (11) ensures that borrow sites blocks are moved to only one location if used at all. Constraint (12) ensures that waste site blocks are filled by a single cut block if used at all. Constraint (13) ensures that the correct material is hauled between blocks, i.e. if $\omega_b = \omega_{b'}$ then $H_{b,b'} \geq 0$. Constraint (14) is an alternative constraint to (13) that utilises the binary parameter $\kappa_{b,s}$. Constraint (15) ensures that borrow site blocks are not moved to waste site blocks. Constraint (17) ensures that the decision variable is binary. Constraint (16) is a special precedence constraint and ensures that both binary parameters can have a value of zero, or one may be non-zero, but both cannot take the value one. More precisely, if $(z_a > z_b)$ and $(z_c < z_d)$ then it is not permissible for $H_{a,d} = 1$ and $H_{b,c} = 1$. Similarly if $(z_a < z_b)$ and $(z_c > z_d)$ then it is not permissible for $H_{a,d} = 1$ and $H_{b,c} = 1$. These conditions can be translated into the following set:

$$\wp = \{(a,b,c,d) | (a,b) \in B^-_{x-y}, (c,d) \in B^+_{x-y}, [(z_a > z_b) \wedge (z_c < z_d)] \vee [(z_a < z_b) \wedge (z_c > z_d)]\}$$

where $B^-_{x-y} = \{(b,b') \in B^- \cup \hat{B}^- | (x_b = x_{b'}) \wedge (y_b = y_{b'})\}$ and $B^+_{x-y} = \{(b,b') \in B^+ \cup \hat{B}^+ | (x_b = x_{b'}) \wedge (y_b = y_{b'})\}$. Constraint (16) is novel as it takes into account a precedence issue without the explicit definition of sequencing variables. The requirement for a constraint of this sort is now demonstrated. For instance under the assumption that block material is not temporarily set aside and double handled, cutting and filling precedence's may make some cut-fill assignments infeasible. Consider the allocations made in Figure 3. The allocation in Figure 3a) is impossible since b3 can't be filled with material from b1 until b4 is filled using material from b2. Otherwise a bottom up filling condition is violated. As b2 is below b1 it can't be removed straightaway thus making the resource allocation infeasible. The only way to make this resource allocation feasible is to place material from b1 somewhere, cut out b2 and move to b4, pick up material from b1 and then move to b3! The allocation in Figure 3(b) however is feasible because both top-down cutting and bottom-up filling precedence's are satisfied.

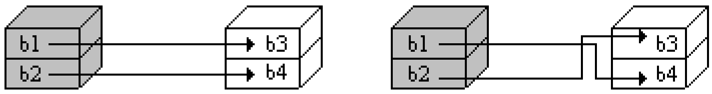


Figure 3. Infeasible and feasible cut-fill allocations (Grey – Cut, White – Fill)

It should also be mentioned that instead of dividing waste and borrow sites into numerous smaller blocks and modelling the movement of material to and from these blocks, one larger block may be defined for each waste or borrow site. This makes block definition and parameter calculation simpler. Let $\aleph_b$ be the number of blocks within a waste site or borrow site *b*. The following constraints then replace constraint (11) and (12):

$$\sum_{b' \in (B^+ \cup \hat{B}^+)} (H_{b,b'}) \leq \aleph_b \quad \forall b \in \hat{B}^- \tag{11b}$$
$$\sum_{b' \in (B^- \cup \hat{B}^-)} (H_{b',b}) \leq \aleph_b \quad \forall b \in \hat{B}^+ \tag{12b}$$

These constraints allow waste and borrow site to have $\aleph_b$ blocks added or removed.

As a last remark it should be mentioned that there are many benefits to the above block model, however, the expansion or contraction of excavated material may cause differences in block volumes that are significant in practice. In other words the filling requirement of a block may not be satisfied in some situations but will be exceeded in others. At present the extent of this issue is unknown, but fortunately a solution does exist. Unlike the aforementioned section model and the block model in the next section, incorporating this aspect is less straightforward, and does not involve an alteration to the mathematical model per se. Instead an alteration to the original block data is required. For example the volume (i.e. of cut or fill) at each grid location (x_b, y_b, z_b) should be scaled by the appropriate swell/contraction value first and then partitioned into whole blocks. Consequently the problem returns to that of cutting, moving and filling whole blocks.

3.3. *Block Allocation Model 2*

In second block model the assumption of uniform block material and movement of whole blocks is relaxed. The decision variable for this model is denoted by $Q_{b,b',s}$ and describes the volume to be cut from block b and moved to block b' of soil type s. The binary decision variable $H_{b,b'}$ is no longer strictly necessary. The following block optimization model is therefore proposed:

Minimise $EM = \Omega \times \sum_{b\in(B^- \cup \hat{B}^-)} \sum_{b'\in(B^+ \cup \hat{B}^+)} \sum_s \left(\lceil Q_{b,b',s} / \hat{Q}_{b,b',s} \rceil \times f_{b,b'} \right)$
Subject to:

$$\sum_{b'\in(B^+ \cup \hat{B}^+)} \left(Q_{b,b',s} \right) = cut_{b,s} \qquad \forall b \in B^-, \forall s \tag{18}$$
$$\Upsilon_s \sum_{b'\in(B^- \cup \hat{B}^-)} \left(Q_{b',b,s} \right) = fill_{b,s} \qquad \forall b \in B^+, \forall s \tag{19}$$
$$\sum_{b'\in(B^+ \cup \hat{B}^+)} \left(Q_{b,b',s} \right) \leq cut_{b,s} \qquad \forall b \in \hat{B}^-, \forall s \tag{20}$$
$$\Upsilon_s \sum_{b'\in(B^- \cup \hat{B}^-)} \left(Q_{b',b,s} \right) \leq fill_{b,s} \qquad \forall b \in \hat{B}^+, \forall s \tag{21}$$
$$Q_{b,b',s} \geq 0 \qquad \forall s, \forall b \in \left(B^- \cup \hat{B}^- \right), \forall b' \in \left(B^+ \cup \hat{B}^+ \right) \tag{22}$$
$$Q_{b,b',s} = 0 \qquad \forall s, \forall b \in \hat{B}^-, \forall b' \in \hat{B}^+ \tag{23}$$
$$Q_{b,b',s} = 0 \quad \forall s, \forall b \in \left(B^- \cup \hat{B}^- \right), \forall b' \in \left(B^+ \cup \hat{B}^+ \right) | \left(cut_{b,s} = 0 \right) \vee \left(fill_{b',s} = 0 \right) \tag{24}$$

Constraint (18) ensures that all material from cut blocks is moved elsewhere. Similarly, Constraint (19) ensures that all material for fill blocks is obtained from cut blocks. Constraint (20) and (21) are capacity constraints that ensure that disposal and waste blocks are not over utilised. Constraint (22) is a simple positivity constraint for the amount of material moved of each soil type between each pair of blocks. Constraint (23) ensures that no material is moved between borrow and waste sites. Constraint (24) ensures that non-existent material cannot be moved between blocks; this is believed to be a redundant constraint as (18) and (19) should enforce this.

To enforce correct precedence, the binary haul variable of section 3.2 must be reintroduced and the following constraints must be added:

$$H_{a,d} + H_{b,c} \leq 1 \qquad \forall (a,b,c,d) \in \wp \tag{25}$$
$$H_{b,b'} \in \{0,1\} \qquad \forall b \in \left(B^- \cup \hat{B}^- \right), \forall b' \in \left(B^+ \cup \hat{B}^+ \right) \tag{26}$$
$$Q_{b,b',s} \leq H_{b,b'} \times \mathcal{L} \qquad \forall s, \forall b \in \left(B^- \cup \hat{B}^- \right), \forall b' \in \left(B^+ \cup \hat{B}^+ \right) \tag{27}$$

As cut and fill can occur in the same blocks it is necessary to enforce that $a \neq b \neq c \neq d$ for each tuple in set $\wp$. In the event that numerous soil types are acceptable as fill within blocks, constraint (18) and (20) are rewritten as follows:

$$\sum_s \sum_{b'\in(B^- \cup \hat{B}^-)} \left(Q_{b',b,s} \right) = fill_b \qquad \forall b \in B^+ \tag{28}$$
$$\sum_s \sum_{b'\in(B^- \cup \hat{B}^-)} \left(Q_{b',b,s} \right) \leq fill_b \qquad \forall b \in \hat{B}^+ \tag{29}$$

Similarly, the following constraint is required to reject movements of invalid material:

$$Q_{b,b',s} \leq \kappa_{b',s} \mathcal{L} \qquad \forall s, \forall b \in \left(B^- \cup \hat{B}^- \right), \forall b' \in \left(B^+ \cup \hat{B}^+ \right) \tag{30}$$

4. Modelling Fuel Consumption

Fuel consumption is based upon many things, but the distance travelled, the speed of motion, the surface type and the surface gradient are believed predominant. In this section the forces of motion on an inclined plane are compared to those on level ground and fuel consumption metrics are proposed. A review of the physics and mathematics of inclined planes, forces and power should be consulted before reading this section. The main parameter that is used in this section is FOR, the force that retards or enhances motion on an inclined plane. It is taken as the sum of friction, drag and resolved weight and computed as follows: $FOR = Mgsin\,\theta + \mu_f Mgcos\,\theta + 0.5\mu_d A\rho V^2$. It is a function of mass (M), speed (V), cross-sectional area (A) and parameters for drag and friction denoted by μ_d and μ_f respectively, and the density of air (ρ).

Let $\mu = (\mu_f, \mu_d)$ and let **FOR**(θ, μ, M) be a function that returns the force retarding or enhancing motion. It represents the previously defined parameter FOR; therefore **FOR**$(\theta, \mu, M) = FOR$. For ease of reading, the following abbreviation, $\mathbf{FOR}(\theta)$ is used where appropriate; for instance where μ and M are common to all terms. The ratio of $\mathbf{FOR}(\theta)$ and $\mathbf{FOR}(0)$ is denoted by $\boldsymbol{\gamma}(\theta)$ and indicates how much harder or easier it is to travel at an angle than it is to travel on level ground. As work is equal to force multiplied by distance, then the ratio of these two values is also the ratio of the work done over a distance D. It should be noted that work is a measure of the power consumption over a given time. Therefore if D is the distance travelled then work is defined as follows: $\mathbf{W}(D, \theta, \mu, M) = D \times \mathbf{FOR}(\theta, \mu, M)$ and is a metric for the total effort of movement.

When travelling at an angle the instantaneous fuel consumption (IFC) is conceptually proportional to the power consumption of the vehicles engine (i.e. P) in response to the forces against motion, i.e. $\mathbf{IFC}(\theta) \propto \mathbf{FOR}(\theta)$. It is also proportional to the IFC when travelling level. Therefore $\mathbf{IFC}(\theta) = \boldsymbol{\gamma}(\theta) \times \mathbf{IFC}(0)$.

Proof: As $\mathbf{IFC}(\theta) = P(\theta) = \mathbf{FOR}(\theta) \times V$ and $\mathbf{IFC}(0) = P(0) = \mathbf{FOR}(0) \times V$ then $\boldsymbol{\gamma}(\theta) \times \mathbf{IFC}(0) = \frac{\mathbf{FOR}(\theta)}{\mathbf{FOR}(0)} \times \mathbf{FOR}(0) \times V = P(\theta) = \mathbf{IFC}(\theta)$.

It is proposed that the fuel consumption when travelling at an angle be approximated by the following equation: $\mathbf{FC}(\theta) = D \times \mathbf{FOR}(\theta)$. The reason for this is as follows. Fuel consumption is the instantaneous fuel consumption per unit of time multiplied by the time of operation, i.e. $\mathbf{FC}(\theta) = \mathbf{IFC}(\theta) \times T = \mathbf{IFC}(\theta) \times \frac{D}{V}$. As $\mathbf{IFC}(\theta)$ is proportional to $\mathbf{IFC}(0)$ then $\mathbf{FC}(\theta) = \mathbf{IFC}(\theta) \times \frac{D}{V} = \boldsymbol{\gamma}(\theta) \times \mathbf{IFC}(0) \times \frac{D}{V}$. Similarly as $\mathbf{IFC}(0) = \mathbf{FOR}(0) \times V$ then $\mathbf{FC}(\theta) = \boldsymbol{\gamma}(\theta) \times \mathbf{FOR}(0) \times D$. Substituting for $\boldsymbol{\gamma}(\theta)$ provides the final result.

If λ_M denotes the number of litres consumed per unit of distance on level ground for a vehicle of mass M, then the following equation is a proxy for fuel consumption:

$$\mathbf{FC}(D, \theta, \mu, M, \lambda_M) = \lambda_M \times \boldsymbol{\gamma}(\theta) \times D$$

In the above equation $\boldsymbol{\gamma}(\theta)$ is used as a mechanism to rescale λ or D (but not both). Hence when travelling at an angle, it is equivalent to travelling over level ground over another distance (i.e. greater or smaller). Similarly the number of litres consumed per unit of distance is also greater or smaller. Let μ_{fc} be a coefficient of fuel consumption that varies for different vehicles. Then the following equations could also provide an accurate estimation of fuel consumption:

$$\mathbf{FC}(D, \theta, \mu, M, \lambda_M) = \mu_{fc} \times \mathbf{FOR}(\theta) \times D$$
$$\mathbf{FC}(D, \theta, \mu, M, \lambda_M) = \mu_{fc} \times \lambda_M \times \boldsymbol{\gamma}(\theta) \times D$$

Preliminary testing has shown that the following relation, 1 Joule $\propto 1 \times 10^{-6} = 0.000001$ litres, is a reasonable approximation to the truth. Hence, $\mu_{fc} = 0.000001$.

5. Case Study and Numerical Investigations

The approaches developed in this article are now applied. In particular the proposed models are tested and compared. OPL Studio (also known as CPLEX) was used to solve them on a quad core, Dell PC with a 2.5 GHz processor and 4GB memory. The majority of the model parameters (i.e. distances, gradients, forces, fuel consumption, etc) were computed using some basic procedures that were coded the C++ programming language. The following factors were also are considered:

- Soil type: One (general), Two (suitable, unsuitable), Four (Class A (top soil | organic), Class B (sand | clay | gravel | silt), Class C (weathered rock | thinly bedded), Class D (hard rock)). Class A is regarded as unsuitable as common fill.
- Models: Section, Block_1, Block_2
- Metrics: work, distance
- Block Size $(\Delta x, \Delta z)$: (20,1), (50,1).

The general soil type category includes all earth types; in other words there is no differentiation in type. With regard to approximating fuel consumption it is assumed that 1 Joule ≡ 1E-6 Litre, i.e. 1 Litre ≡ 1E6 Joules ≡ 1000 kJ. It is also assumed that fuel costs $1.50 per litre. The parameters used to calculate forces are as follows, $(\mu_d, \mu_f, \rho) = (1, 0.01, 1.2)$. The vehicles used to perform the hauls and the conditions of their use are shown in Table 1.

Table 1. Vehicle specifications

Vehicle	Mass (kg)	Speed (m/s)	Cross Sectional Area (m^2)	Capacity (m^3)	Power (kW)	Condition of use
Dozer	100000	4	10	10	700	≤ 50m
Scraper	65000	12	6	20	400	51 – 1500 m
Truck (On Site)	70000	17	10	35	400	1501 – 3000 m
Truck (Off Site)	70000	20	10	30	300	> 3000m

The case study is a road construction project of length 7 km and was obtained from the Queensland Department of Transport and Main Roads (QDTMR). It contains elevation data from a real road construction project in northern Queensland (Australia). The case study data has been made available in Burdett and Kozan (2013). It should be noted that the original problem has been enlarged somewhat by inverting, appending and altering elevation values. This approach was taken as a suitably large sized and comprehensive case study was not available. Many road projects are quite short and imbalanced in urban settings, leading to a large amount of excess material, or a large amount of borrowed material, and little reshaping of the terrain. Issues with incomplete and inaccurate data, and confidentiality also affected this choice.

The terrain and planned road profiles are shown in Figure 4. Data occurs every 50 metres and the road width is 20 metres. The earthwork volumes are based purely on the longitudinal profiles and the land is assumed to follow the line connecting adjacent elevations. Volumes of batters and benches are not included in this case study but could have been, were they available.

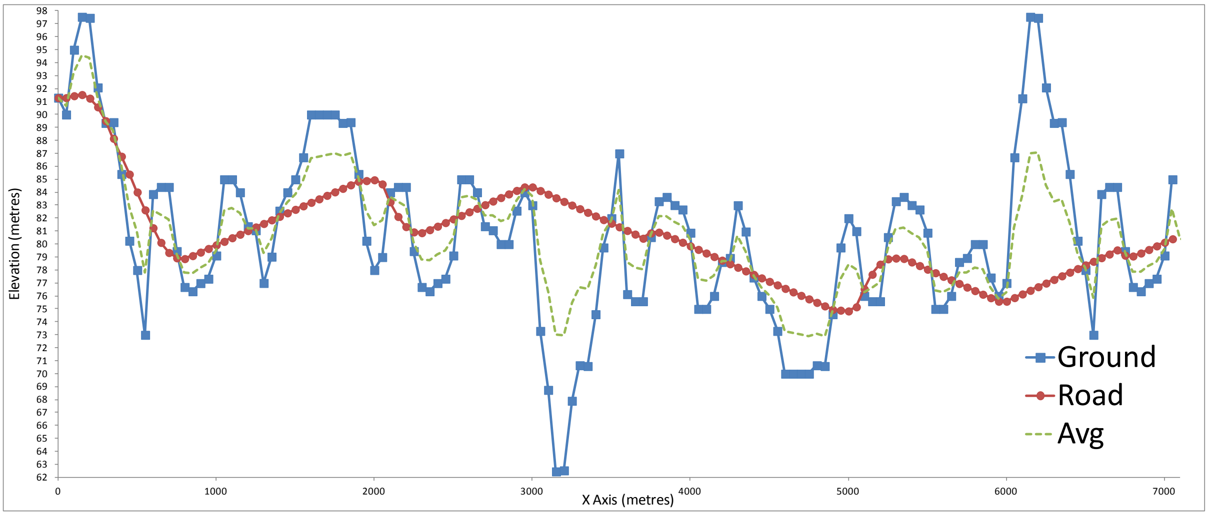


Figure 4. Longitudinal profiles of ground and planned road surface (data occurs every 50 metres)

There were 143 sections. When intersections points are included, and used as section boundaries, the number of sections increased to 172. Of this number, 85 involved cuts and 87 involved fills. The net cut and fill required was about 298,105 $m^3$and 293,120 m^3 respectively, i.e. an excess of 4985 cubic metres of cut. After repartitioning sections, the net cut and fill required was 304,232 and 299,247 cubic metres respectively; still a difference of 4985 m^3 as required. Different net cut and fill values occur because the cut and fill within the same section are coalesced initially, but after the repartitioning this does not occur.

In cut sections, the percentage of material that was suitable for fill has been generated arbitrarily. This was necessary as exact soil type information was unavailable. For the purposes of this article it has been assumed that most material is suitable as fill and hence the percentages have been chosen between 80 and 100% (although other values could have been selected). In practice all percentages are feasible. Furthermore these values are site dependent and could vary greatly. The suitable and unsuitable material was also assumed to occur uniformly within each section in order to simplify data generation for the block models. This may be different in practice and would otherwise have been provided. The uniform occurrence of unsuitable material within sections was also used as a mechanism to allow a direct comparison between the section-based model and the block models. The position of unsuitable material within sections is not utilised by the section based model and is a limitation of that approach, particularly when sections are large.

The total amount of suitable and unsuitable material respectively (within cut sections) was 278,797 m^3 and 25,435 m^3. After unsuitable material was removed, there was a shortage of about 20,450 m^3 of fill. This must be obtained off site. One borrow site was defined to the right of the project site, at a distance of 30 km. Several waste sites were defined for the unsuitable material. These were located at either end of the project site. In practice, waste material is often stored at the end of a project site, and resumed at a future time, for other road projects.

The volume of cut and fill within each section is further decomposed into amounts from each category (i.e. Class A, B, C and D). In cut sections, these volumes must be used as fill or hauled as waste. The volume of cut material in each section has been generated randomly: 0-20% for class D, 0-40% for class C, and class B is whatever is left over. All unsuitable material is assumed to be solely Class A. The volume of fill material in each section is assumed to be 50% class B, 30% class C, and 20% class D. In fill sections, these volumes are strictly required. A summary of the different volumes is shown in Table 2.

Table 2. Material requirements and surplus/deficits in m^3

Material	Class A	Class B	Class C	Class D
Available (From Cuts)	25435.08	196561.10	54228.37	28007.30
Required (as Fill)	0.00	149623.41	89774.04	59849.36
Deficit (Borrow)	0.00	0.00	35545.67	31842.06
Surplus (Waste)	25435.08	46937.69	0.00	0.00

Given the deficits in Table 2, the borrow site capacity was increased to 68,000 m^3. The section based model was solved using CPLEX and the results are summarised in Table 3. The time to solve the model was minimal in each case, i.e. no more than a few seconds. The section based model is a traditional LP. To solve it the Dual Simplex algorithm was used by CPLEX. A pre-solve stage however was first applied. The pre-solve stage eliminated 14792 rows and 7400 columns for the one soil problem. The reduced LP had 172 rows, 7393 columns and 14786 non-zeros. For the two soil problem, the pre-solve eliminated 29668 rows and 22022 columns and did 84 substitutions. The reduced LP had 174 rows, 7477 columns and 14954 non-zeros. For the four soil problem, the pre-solve eliminated 59260 rows and 37832 columns and also did 84 substitutions. The reduced LP had 506 rows, 21247 columns, and 42494 non-zeros.

Table 3 shows that the work based metric provides solutions are quite comparable in terms of distance but are superior in terms of cost and fuel consumption. The difference is roughly 15000 litres or about $20000. The solutions are quite different when compared graphically, particularly between the general situation (one soil type) and the others, which have numerous. Obviously the more constraints there are on what material may be moved where (i.e. meant literally and not mathematically), the more difficult the problem becomes, and the higher the cost of earthworks. For example, if a volume of soil is not suitable as fill in a nearby location, then it may need to be moved to another location at greater distance and hence cost. Similarly if there is not enough soil of particular types, then borrow sites may need to be utilised – again at greater cost if they are not adjacent to the project site. These situations are well reflected in Table 3.

Table 3. Results of section based model

Soil Types	Metric	Work (J)	Distance (km)	Fuel Estimate (Litre)	Cost Estimate ($)
1	Work	1.03559E+11	13,948	103,559	155,338
1	Distance	1.15252E+11	12,668	115,252	172,878

Soil Types	Metric	Work (J)	Distance (km)	Fuel Estimate (Litre)	Cost Estimate ($)
2	Work	2.24727E+11	26,206	224,727	337,091
2	Distance	2.3393E+11	25,205	233,930	350,895

Soil Types	Metric	Work (J)	Distance (km)	Fuel Estimate (Litre)	Cost Estimate ($)
4	Work	4.92449E+11	55,753	492,449	738,674
4	Distance	5.08301E+11	54,024	508,301	762,452

To apply block model one, the assumption of full blocks was made. The accuracy of the solution depends on the block size and Table 4 shows the difference, for this case study, for different block sizes. For example a 1 by 1 block (in the X-Z) plane results in 67% of blocks being full (i.e. 24906 blocks) and an average block content of 81% (i.e. 16.2 m^3 out of 20). For demonstrative purposes, the block partition for blocks of length 50m and height 1m (i.e. $\Delta x = 50, \Delta z = 1$) are shown in Figure 5. This was created in Microsoft Excel using a generic VBA macro.

Table 4. Effect of block size on net cut/fill in blocks

Block Size ($\boldsymbol{\Delta x}, \boldsymbol{\Delta y}, \boldsymbol{\Delta z}$)	Block Volume ($\boldsymbol{m^3}$)	Number Blocks	%Full (Avg)	%blocks in category (0-10%, 11-20%, ..., 91-100%) full
(1, 20, 1)	20	37,173	81.17	(4.32, 3.56, 3.39, 3.33, 4.52, 4.41, 3.07, 2.81, 3.44, 67.15)
(10, 20, 1)	200	3890	77.5	(7.4, 3.98, 3.47, 3.5, 4.83, 4.27, 2.78, 2.7, 4.22, 62.85)
(20, 20, 1)	400	2040	73.85	(9.41, 5.29, 4.02, 3.53, 4.56, 3.48, 3.87, 3.73, 3.92, 58.19)
(50, 20, 1)	1000	950	63.09	(3.65, 1.8, 1.26, 1.34, 1.41, 1.26, 0.85, 1.13, 1.26, 10.46)

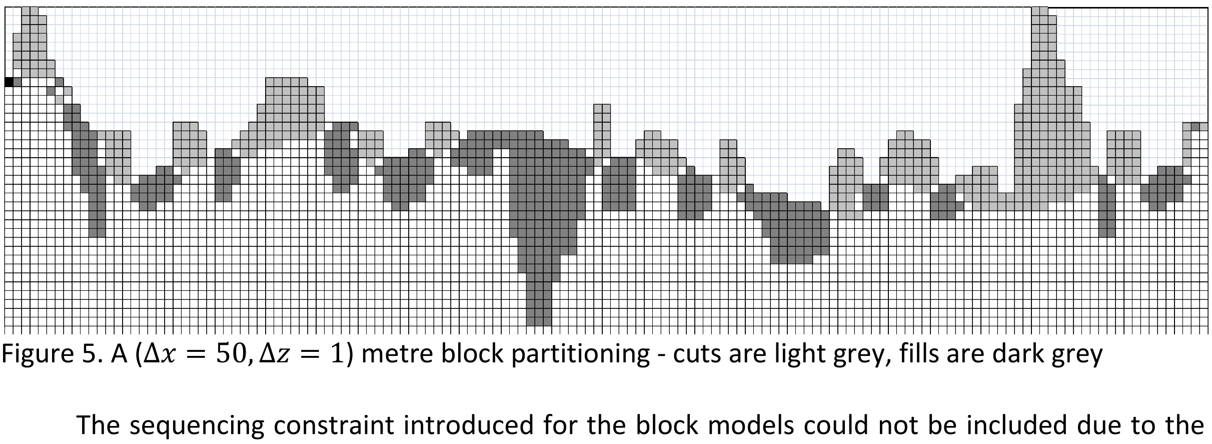


Figure 5. A ($\Delta x = 50, \Delta z = 1$) metre block partitioning - cuts are light grey, fills are dark grey

The sequencing constraint introduced for the block models could not be included due to the size of the problems considered. For example, for the 953 (i.e. 50×1) block situation there were 7,816,524 tuples in set $\wp$. Consequently the model required 7,816,524 additional constraints. It was impossible to generate this set in OplStudio. It was similarly impossible to load this information from a text file, due to insufficient memory, when generated for instance in C++.

The first block model was solved and the results are shown in Table 5. It should be noted that a smaller block size of $(\Delta x = 10, \Delta z = 1)$ was also investigated which resulted in 3893 blocks. OplStudio however was unable to solve the model in a conventional way as there was insufficient PC memory.

The first block model is a mixed integer programming (MIP) problem. CPLEX solved this model using an MIP search method called dynamic search. The emphasis of that method was to balance optimality and feasibility. For the larger 20 metre block size, the reduced MIP had 2074 rows, 1075340 columns, 2150680 non-zeros, and 1075340 binaries. The smaller 50 metre block size resulted in a reduced MIP with 986 rows, 243043 columns, 486086 non-zeros, and 243043 binaries.

Table 5. Results of Block-1 model

Block Size $(\boldsymbol{\Delta x}, \boldsymbol{\Delta z})$	# Block	Soil Types	Metric	Work (J)	Distance (km) (Recti-linear)	Fuel Estimate (Litres)	Cost Estimate ($)
(20,1)	2043	1	Work	4.4590E+10	16,736	44,590	66,885
			Distance	6.6157E+10	15,138	66,157	99,235
(50,1)	953	1	Work	6.6041E+10	21,197	66,041	99,062
			Distance	8.9043E+10	19,608	89,043	133,564

Table 5 shows that superior solutions are obtained when a smaller block size is used. This table also shows that "work" and "distance travelled" are somewhat smaller than those obtained by the section based model. It has been theorised that this occurs because a direct path between blocks is taken, whereas the section based approach takes into account the existing terrain and passage through it (i.e. the ups and downs between two locations) in an approximate way. The resulting fuel consumption and costs would therefore be smaller. To better compare section and block models, the path between blocks should also be approximated in the same way that paths between sections were; for example as the sum of movements across several "in-between" sections. Each stack of blocks is therefore regarded as a section with an average height: $h_i = dz \times 0.5 \times \left(\max_{\forall b|x_b=i} z_b + \min_{\forall b|x_b=i} z_b\right)$ where *i* is a grid location in the x-axis. Therefore the path between two adjacent stacks is from the stack middle points and the work required and distance of travel between any two blocks is as follows:

$$D_{b,b'} = \sum_{i=x_b,..,x_{b'}} \mathbf{ED}\big(i.dx, h_i, (i+\psi)dx, h_{i+\psi}\big) \tag{31}$$

$$W_{b,b'} = \sum_{i=x_b,..,x_{b'}} \mathbf{ED}\big(i.dx, h_i, (i+\psi)dx, h_{i+\psi}\big) \times F_T(\theta_i) \tag{32}$$

In the above equations theta is $\theta_i = \tan^{-1} G_i$ and the gradient is $G_i = \left(h_{i+\psi} - h_i\right)/dx$, **ED** is a function to compute the Euclidean distance between two points (x_1, y_1, x_2, y_2), and F_T is the function to compute the force on an incline of angle theta. The model was re-applied and the new results are shown in Table 6. Table 6 shows that the solution quality has deteriorated somewhat, however it is still quite comparable to the results shown in Table 5. For example, the results are of the same order of magnitude. Therefore the use of an indirect approximation of distance travelled and work has not resulted in significantly poorer solutions. In comparison to the section based approach, the solutions of the first block model are quite superior. It can be concluded that the modelling of elevation is significant. It should be noted that each of the problems in Table 5 and 6 has a large number of binary decision variables. For example, for the smaller 20 metre block size, there were over 1 million.

Table 6. Results of revised Block-1 model

Block Size $(\boldsymbol{\Delta x}, \boldsymbol{\Delta z})$	# Block	Soil Types	Metric	Work (J)	Distance (km) (Recti-linear)	Fuel Estimate (Litres)	Cost Estimate ($)
(20,1)	2043	1	Work	5.5124E+10	16,670	55,124	82,685
			Distance	6.6870E+10	15,414	66,870	100,305
(50,1)	953	1	Work	9.3547E+10	21,405	93,547	140,321
			Distance	9.8323E+10	19,975	98,323	147,484

To apply the second block model, the volume of suitable and unsuitable material in each block was computed by "equating" the volumes and percentages of suitable and unsuitable material previously defined within sections. To do this, the relative position of each block was identified in relation to the specified sections. Generally a block can be wholly contained or it may over lap two sections. Blocks that overlap three or more sections have not resulted and hence have not been considered in this article. A simple formula computes the percentage of the blocks content that exists within a section and this percentage is then multiplied by the blocks original volume. Given the assumption that soil types occur uniformly across both sections and blocks, the aforementioned volume is then divided amongst the different categories of material, i.e. by multiplying by the appropriate percentage.

The second block model was solved and the results are shown in Table 7. The second block model is a traditional LP buts its size is quite large. To solve it the Primal Simplex algorithm was used by CPLEX. A Cholesky factorisation was also utilised by CPLEX. For the larger 20 metre block size, and one soil type, the pre-solve stage eliminated 1075383 rows and 24 columns. The reduced LP had 2074 rows, 1075340 columns, and 2150680 non-zeros. A compression algorithm was also used to reduce the A matrix from 119.8 Mb to 23.54 Mb. For the 20 metre block size and two soil type, the pre-solve stage eliminated 3225252 rows and 1073426 column. The reduced LP had 2076 rows, 1076319 columns, and 2152638 non-zeros. The compression stage reduced the A matrix from 260 Mb to 46 Mb.

The third column in Table 7 refers to the number of cut-fill pairings in the solution. It is not the total number of trips required to perform the hauls. The values in this column do not differentiate between soil types either. Otherwise the number of hauls would be larger. Table 7 again shows that the cost of earthworks becomes increasingly more expensive when taking into account more soil types and constraints upon where they can and can't be used. The work based metric also provides superior solutions in terms of cost. The extent of this improvement depends greatly upon the parameters that were selected and could be significantly greater in other situations. The results in Table 7 are not as good as those obtained by the section based model. For four soil types the results are also somewhat higher. There is also less difference in the solution quality between the different blocks sizes. This occurs because the second block model is able to divide block material and to re-distribute to multiple destinations; a feature not available to the first block model.

Table 7. Results of Block-2 model

Block Size $(\Delta x, \Delta z)$	Soil Types	# Hauls	Metric	Work (J)	Distance (km)	Fuel Estimate (Litres)	Cost Estimate ($)
(20,1)	1	1987	Work	5.37224E+10	13,906.70	53,722.4	80,583.60
(20,1)	1	1987	Distance	5.93186E+10	12,961.003	59,318.6	88,977.90
(50,1)	1	969	Work	5.42708E+10	13,825.79	54,270.08	81,406.20
(50,1)	1	969	Distance	5.70472E+10	12,930.97	57,047.20	85,570.80

Block Size $(\Delta x, \Delta z)$	Soil Types	# Hauls	Metric	Work (J)	Distance (km)	Fuel Estimate	Cost Estimate ($)
(20,1)	2	3047	Work	2.49909E+11	35,487.25	249,909	374,863.50
(20,1)	2	3047	Distance	2.54671E+11	34,559.15	254,671	382,006.50
(50,1)	2	1434	Work	2.51130E+11	35,552.57	251,130	376,695
(50,1)	2	1434	Distance	2.53604E+11	34,647.49	253,604	380,406

Block Size $(\Delta x, \Delta z)$	Soil Types	# Hauls	Metric	Work (J)	Distance (km)	Fuel Estimate	Cost Estimate ($)
(20,1)	4	-	Work	7.1914E+11	87,237.06	719,140	1,078710
(20,1)	4	-	Distance	7.2344E+11	86,460.50	723,440	1,085,160
(50,1)	4	2954	Work	7.33430E+11	88,655.34	733,430	1,100,145
(50,1)	4	2964	Distance	7.35184E+11	88,027.50	735,184	1,102,776

Testing the validity of solutions is more difficult when dealing with blocks. In the section based approach, several chart types, that use arrows, clearly show where material is moved to, and whether these movements are correct and reasonable. Congestion on these charts is typically minimal. Due to the large number of blocks and the partitioning of the terrain vertically, these approaches are no longer useful. The level of congestion on these charts is great and much detail is hidden. An animation of blocking cutting and filling is a substitute approach that is recommended. It is quite easy to encode in Excel/VBA and shows a solution in sufficient detail when watched in entirety. A prototype animation tool has been developed for testing the validity of solutions and for demonstrating solutions to third parties. Screen shots of this tool are available from the authors upon request.

In collating results for Table 7 it has been found that as the number of soil types increases, and the block size decreases, then the block models become increasingly more difficult to solve due to PC memory limitations. This is because the number of decision variable is directly related to these factors. The problem sizes were on the limit of the PC's memory, and careful encoding of the model was needed, for example by removing superfluous variables and parameters. For some of the larger problem instances this was not sufficient and different LP solvers and options had to be used as opposed to the default options. These were selected by trial and error as the CPLEX IDE provides little documentation concerning which solvers, tolerances and options should be used. Even so, the 20 metre block length problem with four soil types could not be solved. The total memory requirement for this problem is at least 8GB which is twice the capacity of the PC that was available for this numerical investigation. However, the model could be solved one soil type at a time and then reassembled. This is because the decision variables and constraints for each soil type were independent of other soil types and borrow and waste sites had adequate capacity.

6. Conclusions

This article has considered how to construct more detailed and hence more accurate strategic earthwork allocation plans for linear infrastructure projects. The article has also considered the environmental impact of construction activities. A significant improvement over previous approaches has been provided because fuel consumption and terrain gradients have been explicitly incorporated. Several metrics were proposed to quantify fuel usage and hence emissions. These are integrated into the proposed LP and MIP optimization models.

This article is also significant and innovative for a number of other reasons. In the proposed section based approach, an average ground profile is suggested to better approximate the distance and gradients of real hauls. This is achieved without directly modelling the changing terrain over time. An approach however that does explicitly model the changing terrain over time would be novel.

The partitioning of the project site into discrete blocks (i.e. containers of earth) and the development of the associated block models is an approach yet to be reported in the literature. The proposed block models are superior to a section based approach as they model the position of earth at different elevations and provide additional decision making opportunities that would otherwise have been unavailable. Haulage costs can also be modelled more accurately because of the added structure provided by the blocks. The block models are generic and are readily applicable for both 2D and 3D scenarios. The distance between sections and blocks is not direct, i.e. it is the sum of separate movements over inclined planes of different angles. This approach is superior to those taken in other articles as it more accurately models the movement of material through the terrain as it is altered (i.e. instead of defining costs according to a direct path).

The physics concept work has not been used in earthworks planning before to our knowledge. As a proxy for fuel consumption, it provides a powerful generic metric. Although it is simpler to use than other more accurate but complex variants found in the literature, the primary motivation for its use is that an earthwork allocation is a plan of one aspect of earthwork activities and often does not need to include the specific types of vehicles that will be used in the project; besides these are often unknown and or variable. In practice there seems to be little point in creating an earthwork allocation based upon the fuel consumption of one fleet of vehicles if the fleet is changed numerous times before construction activities end. Our approach however does facilitate the option to model fuel consumption accurately for specific vehicles. Obviously the planner decides which approach is required on each project, based upon what they hope to achieve and how much effort they are willing to apply.

In this article the solution of a 2D road construction case study has been concentrated upon. The numerical results have shown that the use of work as a proxy for fuel consumption is very reasonable. The reduction in fuel usage depends upon the specified conversion of energy (in Joules) to litres of fuel consumed. In the absence of accurate information, this relationship has been approximated. However the results of this article are indicative of a real potential for significant savings and a motivator for continued investigation and analysis. The terrain also affects the solution greatly. As a single case study has been investigated it is impossible to generalise what improvement is typical. It is suggested that a large number of different case studies be investigated. Given the difficulty in obtaining data for case studies this is expected to be a difficult and time consuming task, one well outside the scope of this article.

The numerical investigations have mimicked the effect of different planning scenarios. The three problems that were solved reflect the different scenarios (and perhaps phases) that occur in practice. For example at the preliminary earthworks planning phase, only rough terrain information may be known and nothing is really known about what specific type of material exists under the ground. Later an estimate of how much material is suitable and unsuitable can be approximated and a preliminary earthworks plan can then be developed. Finally, a reasonable estimation of what soil types are present and in what quantities can be identified and utilised to provide a more accurate and superior plan. Hence the three scenarios represent the 1 soil type, 2 soil type and many soil type problems. The numerical results show that the cost of earthworks could become increasingly more expensive when taking into account more soil types and constraints upon where they can and can't be used. The difference in cost can be great and demonstrates that preliminary earthwork plans can be quite poor at taking into account the real costs. This result provides ample motivation for more preparations initially, for example in determining soil type quantities, so that superior plans can be constructed and unforseen construction costs are not accumulated.

The ease with which the blocks models can be solved using traditional "off the shelf" mathematical techniques is a potential issue to practitioners as the number of decision variables and constraints increases greatly when the number of blocks and the number of soils types is increased. At present some model instances cannot be solved easily on a PC using commercial software however the use of computers with greater specifications (i.e. memory) overrides many of these issues. A solution to this issue is simply to use alternative solution techniques. In future, other techniques may be needed, such as graph theoretic, meta-heuristics, column generation, and dynamic programming. In the numerical investigation the sequencing constraint also had to be removed and should be included for complete realism. Techniques to correct an infeasible allocation (with respect to sequencing) have been developed but this topic is outside the scope of the article.

An ordering of block cutting, hauling and filling activities can be made; this provides a real plan that can be used in practice. From this sequence a real schedule of activity timings can also be constructed. This is a topic for future research and for another article. The integration of earthwork planning and machine selection also seems to be a very necessary and worthwhile avenue for future work as the choice of machinery greatly affects the merit of making certain cut to fills.

Acknowledgements

This work was performed as part of an ARC linkage grant (LP110200314) in conjunction with Queensland University of Technology, Swinburne University and the Sustainable Built Environment National Research Centre (SBENRC).

References

Akay, A.E. 2006. Minimizing total costs of forest roads with computer-aided design model. *Sadhana*, 31(5),

621–633.
Akcelik, R. 1989. Efficiency and drag in the power-based model of fuel consumption, *Transportation Research B*, 23, 376-385.
Akcelik, R., R. Smit, and M. Besley. 2012. Calibrating fuel consumption and emission models for modern vehicles. IPENZ Transportation Group Conference, Rotorua, New Zealand.
Ammar, M. A., A. I. Eldosouky, and F. A. Jarad. 2003. Optimization of earthwork allocation with multiple soil types. *Alexandria Engineering Journal*, 42(1), 111-125.
Aruga, K., J. Sessions, and A. E. Akay. 2005. Heuristic planning techniques applied to forest road profiles. *Journal of Forest Research*, 10(2), 83-92.
Avetisyan, H.G., E. M. Hooks, and S. Melanta. 2012. Decision models to support greenhouse gas emissions reduction from transportation construction projects. *Journal of Construction Engineering and Management*, 138, 631 -641.
Burdett, R., and E. Kozan. 2013. Earthworks planning for road construction projects: a case study. In Kajewski, S.L., Manley K., & Hampson K.D. (Eds.) Proceedings of the 19th CIB World Building Congress, Brisbane 2013: Construction and Society, QUT Conference. Proceedings, Brisbane Convention Centre.
Burdett, R., and E. Kozan. 2013. Improved earthwork allocation planning for fuel consumption and emissions reduction in linear infrastructure construction – case study data. *QUT Eprints*.
Carmichael, D. G., E. H. Williams, and A. S. Kaboli. 2012. Minimum operational emissions in earthmoving. *In ASCE Construction Research Congress*, 21-23
Dawood, N. N., and S. Castro. 2009. Automating road construction planning with a specific-domain simulation system. *Journal of Information Technology in Construction*, 14, 556-573.
Goktepe, A. B., A. H. Lav, S. Altun, and G. Altintas. 2008. Fuzzy decision support system to determine swell/shrink factor affecting earthwork optimization of highways. *Mathematical and Computational Applications*. 13(1), 61-70.
Hola, B., and K. Schabowicz. 2010. Estimation of earthworks execution time-cost by means of artificial neural network. *Automation in Construction*, 19, 570-579.
Ji, Y., A. Borrmann, E. Rank, F. Seipp, and S. Ruzika. 2010. Mathematical modelling of earthwork optimization problems. Proceedings of the 13th International Conference on Computing in Civil and Building Engineering (ICCCBE-XIII). Nottingham, UK.
Ji, Y., A. Borrmann, and J. Wimmer. 2011. Coupling microscopic simulation and macroscopic optimization to improve earthwork construction processes. Proceedings of the EG-ICE Workshop on Intelligent Computing in Civil Engineering. Enschede, The Netherlands.
Karimi, S. M., S. J. Mousavi, A. Kaveh, and A. Afshar. 2007. Fuzzy optimization model for earthwork allocations with imprecise parameters. *Journal of Construction Engineering and Management*, 133(2), 181-190.
Kim, E., K. J. Manoj, P. Schonfeld, and H. S. Kim. 2007. Highway alignment optimization incorporating bridges and tunnels. *Journal of Transportation Engineering*. 133(2), 71-81.
Kim, B., H. Lee, H. Park, and H. Kim. 2012. Greenhouse gas emissions from onsite equipment usage in road construction. *Journal of Construction Engineering and Management*. 138(8), 982-990.
Kim, B., H. Lee, H. Park, and H. Kim. 2012: Framework for estimating greenhouse gas emissions due to asphalt pavement construction. *Journal of Construction Engineering and Management*, 138, 1312-1321.
Lee, Y. S., M. J. Skibniewski, and W. S. Jang. 2009. Monitoring and management of greenhouse gas emissions from construction equipment using wireless sensors. Proceedings of the 26th International Symposium on Automation and Robotics in Construction. Austin, USA. Lewis, P. and Hajji, A., 2012. Estimating economic, energy, and environmental impact of earthwork activities. Construction Research Congress: Construction Challenges in a Flat World. 1770-1779.
Nassar, K., and O. Hosny. 2012. Solving the least cost route cut and fill sequencing problem using particle swarm. *Journal of Construction Engineering and Management*, 138(8), 931-942.
Norgate, T. E., and N. Haque. 2010. Energy and greenhouse gas impacts and mineral processing operations. *Journal of Clean Production*, 18, 266-274.
Post, K., J. H. Kent, J. Tomlin, and N. Carruthers. 1984. Fuel consumption and emission modelling by power demand and a comparison with other models. *Transportation Research A*, 18(3), 191-213.
Queensland Department of Transport and Main Roads (QTMR), Training Courses for Road Construction Supervisors – Earthworks, Form 11AT26 and 11AT23. Queensland Government. Australia (1977).Queensland Roads, Bruce Highway Upgrade, Cooroy To Curra. No 10, March, p6-14 (2011). Web: http://tmr.qld.gov.au/~/media/busind/techstdpubs/Queensland%20roads%20technical%20journal/March%202011/QueenslandRoadsEdition10.pdf.
Shah, R. K., and N. Dawood. 2011. An innovative approach for generation of a time location plan in road

construction projects. *Construction Management and Economics*, 29, 435-448.
Son, J., K. G. Mattila, D. S. Myers. 2005. Determination of haul distance and direction in mass excavation. *Journal of Construction Engineering and Management*, 131(3), 302 – 309.
Zhang, H. 2008. Multi-objective simulation optimization for earthmoving operations. *Automation in Construction*, 18, 79-86.